

Flight Frequency Tool

Background

- Analysts manually realign capacity by changing gauge or frequency based on past performance.
- Pros
 - Improves the schedule most of the time.
 - Requires no complicated software to run or decipher.
- Cons
 - Solutions tend to reinforce past behavior.
 - Difficult to establish if a solution is optimal.
 - Solutions resolve network issues only at localized spots.
 - Time-consuming process leads to limited consideration of new opportunities.

Objectives

- Build a simple optimization tool that selects the most profitable flight frequencies based on system contribution
- Produce flight frequencies between airports assigned to specific equipment types.
- **Implement the model:** Prove it solves within reasonable computation time using current technology.
- **Conduct experiments:** Demonstrate the model makes sensible decisions based on data and assumptions.
- **Evaluate financial impact:** Determine if adopting the model as a planning tool increases profits.

Tool Can Be Used For:

- **Network Adjustment**
 - Reassign fleet mix across the existing network.
 - Drop or add specific segments and airport stations.
 - Optimize within the hard constraint of the currently available fleet.
- **Network Design**
 - Build multiple scenarios for potential new hubs and focus cities.
 - Allow for a large, expansive list of possible new flight segments.
 - Keep available fleet size as a baseline constraint for realism.
- **Fleet Plan**
 - Allow total fleet count to vary to test growth.
 - Define boundaries and parameters for the allowable network.
 - Establish minimum acceptable flight frequencies for city pairs.
 - Evaluate alternative fleet possibilities alongside infrastructure setup costs.
 - Build multiple distinct scenarios to ensure strategic robustness.

Model Assumptions

- Demand for travel varies by day according to a spill mode
 - Demand for each O&D modeled as a gamma distribution
 - Recapture rate is zero
- Demand between O&D can be satisfied through a pre-specified set of paths – nonstop or connections
- Actual fleet counts can be transformed into total fleet utilization measured in block hours
- Single Average Fare for each O&D
- Flight costs do not vary with passenger load

Model Objective and Constraints

- Objective is to maximize profitability = revenue – costs
- Model allocates demand to available capacity
- Decisions
 - For every O&D, how many seats to allocate in each possible path for that O&D
 - Path allocation can be added up for all possible paths for a given O&D, resulting in the O&D allocation
 - For every possible nonstop segment, how many flights to fly with each possible equipment type
- Objective

$$\sum_k r_k (\text{Traffic}(a_k)) - \sum_{i,j,p} c_{ijp} y_{ijp}$$

Model Objective and Constraints

- Spill model is nonlinear
 - Implementation should include its linearization
- Constraints
 - Sum of leg allocations cannot exceed capacity assigned to leg
 - Total block hours for each fleet cannot exceed available block hours
 - Airport-fleet flow conservation
 - Optionally, minimum leg frequencies may be set
 - Optionally, directional symmetry or fleet-directional symmetry

Model Formulation – Sets & Variables

Data

I	set of all stations
L	set of all legs; elements are members of $I \times I$
K	set of all ODs; elements are (possibly all non-loop) elements of $I \times I$
$S(k)$	the set of all allowed routes providing $k \in K$; elements are sets of paths in the directed graph (I, L)
P	the set of all airplane types
r_k	revenue per passenger for $k \in K$
f_k	<u>p.d.f.</u> of demand for $k \in K$
c_{ijp}	cost of flying airplane $p \in P$ on leg $(i, j) \in L$
k_p	number of seats on airplane $p \in P$
h_{ijp}	hours required to fly airplane $p \in P$ on leg $(i, j) \in L$
N_p	flying hours available for airplanes of type $p \in P$
M_{ij}	minimum number of flights on leg $(i, j) \in L$

Decision Variables

a_s	seats allocated to route s
y_{ijp}	number of flights for airplane $p \in P$ on segment $(i, j) \in L$
a_k	seats allocated to OD $k \in K$ across all routes (dummy variable)

Optimization Problem

$$\text{Maximize } \sum_k r_k \left(\int_0^{a_k} x f_k(x) dx + a_k \int_{a_k}^{\infty} f_k(x) dx \right) - \sum_{i,j,p} c_{ijp} y_{ijp}$$

Model Formulation Constraints

subject to

$$a_k = \sum_{s \in S(k)} a_s \quad \text{for all } k \in K \text{ (notational convenience)}$$

$$\sum_{\substack{s \text{ using} \\ (i,j)}} a_s \leq \sum_p k_p y_{ijp} \quad \text{for all } (i,j) \in L \text{ (leg allocations must not exceed leg capacity)}$$

$$\sum_p y_{ijp} \geq M_{ij} \quad \text{for all } (i,j) \in L \text{ with } m_{ij} > 0 \text{ (minimum leg frequency)}$$

$$\sum_{i,j} h_{ij} y_{ijp} \leq N_p \quad \text{for all } p \in P \text{ (maximum fleet operating hours)}$$

$$\sum_i y_{ijp} = \sum_k y_{jkp} \quad \text{for all } j \in I, p \in P \text{ (airport-fleet flow conservation)}$$

$$\sum_p y_{ijp} = \sum_p y_{jip} \quad \text{for all } (i,j) \in L \text{ (directional symmetry, optional)}$$

$$y_{ijp} = y_{jip} \quad \text{for all } (i,j) \in L, p \in P \text{ (fleet-directional symmetry, optional)}$$

$$a_s, y_{ijp} \geq 0 \quad \text{for all } s \in \bigcup_{k \in K} S(k), (i,j) \in L, p \in P$$

$$y_{ijp} \text{ integer} \quad \text{for all } (i,j) \in L, p \in P$$

Linearizing The Objective Function

Linearizing the Objective Function:

For each OD $k \in K$, we construct the u_{kl} , T_{kl} , and m_{kl} as above. For each $l \in \{1, \dots, t_k\}$, our linear approximation to T_k is $T_k - T_{kl} = m_l(a_k - u_{kl})$. Since m_{kl} decreases with l , the least line on each interval defines our linear approximation of T_k . So, if we introduce a decision variable T_k for each $k \in K$, then the following formulation achieves an equivalent linearization of the objective function:

$$\text{Maximize } \sum_k r_k T_k - \sum_{i,j,p} c_{ijp} y_{ijp}$$

subject to

$$\begin{aligned} a_k &= \sum_{s \in S(k)} a_s && \text{for all } k \in K \text{ (notational convenience)} \\ T_k &\leq T_{kl} + m_l(a_k - u_{kl}) && \text{for all } k \in K, l \in \{1, \dots, t_k\} \text{ (traffic must not exceed approximation)} \\ \sum_{\substack{s \text{ using} \\ (i,j)}} a_s &\leq \sum_p k_p y_{ijp} && \text{for all } (i,j) \in L \text{ (leg allocations must not exceed leg capacity)} \\ \sum_p y_{ijp} &\geq M_{ij} && \text{for all } (i,j) \in L \text{ with } m_{ij} > 0 \text{ (minimum leg frequency)} \\ \sum_{i,j} h_{ij} y_{ijp} &\leq N_p && \text{for all } p \in P \text{ (maximum fleet operating hours)} \\ \sum_i y_{ijp} &= \sum_k y_{jkp} && \text{for all } j \in I, p \in P \text{ (airport-fleet flow conservation)} \\ \sum_p y_{ijp} &= \sum_p y_{jip} && \text{for all } (i,j) \in L \text{ (directional symmetry, optional)} \\ y_{ijp} &= y_{jip} && \text{for all } (i,j) \in L, p \in P \text{ (fleet-directional symmetry, optional)} \\ a_s, T_k, y_{ijp} &\geq 0 && \text{for all } s \in \bigcup_{k \in K} S(k), (i,j) \in L, p \in P, k \in K \\ y_{ijp} &\text{ integer} && \text{for all } (i,j) \in L, p \in P \end{aligned}$$

Required Inputs

- Set of Stations
- Set of Legs / Segments
- Pre-specified Allowed Routes (Set of allowed paths for an OD pair)
- Airplane / Equipment Types (Set of airplane types, e.g., 737, 738)
- Total Available Block Hours (Maximum flying hours available for fleet)
- Aircraft Capacities (Number of seats on airplane)
- Block Hours per Leg (Hours required to fly airplane on leg)
- Origin and Destination (O&D) Pairs (Set of all OD pairs)

Required Inputs

- Demand Probability Density Functions (The demand distribution function). A Gamma distribution explains why the objective function uses continuous integration.
- Spill Parameters explains the math in the objective function. Because there is "zero passenger recapture" once a flight is full, the revenue is strictly capped at capacity.
- Revenue per Passenger (Revenue per passenger for OD pair)
- Operating Cost per Leg (Cost of flying airplane on leg)
- Minimum Leg Frequency Standards (The minimum number of flights required on a leg)

Potential Improvements to the Formulation

- Introduce dynamic realism by accounting for passenger behavior, competition and non-linear costs
- Path-sensitive. Passengers strongly prefer nonstop flights over connections, heavily penalizing multi-leg routes.
- Employs a Quality-of-Service Index (QSI) model. Market share shifts based on its frequency, plane types, and competitor (OA) schedules.
- Multi-fare partitioning. Divides demand across multiple fare tiers to capture both high-paying business and discount leisure travelers.
- Non-linear step costs. Uses piece-wise linear functions to capture major fixed setup costs like airport gate leases, station openings, and fleet handling.